Equations for the "*Oscillation Module*"

specific to pTet-circuit

$$pTet \xrightarrow{b_1} FlhDC + pTet \tag{1}$$

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specific to pFlhDC-circuit

$$pFlhDC \xrightarrow{b_2} FlhDC + pFlhDC \tag{2}$$

$$n_3 FlhDC + pFliA \underset{k_{-3}}{\overset{k_3}{\rightleftharpoons}} (FlhDC)_{n_3} \bowtie pFliA \qquad K_3^{n_3} = \frac{k_{-3}}{k_3} \tag{3}$$

$$n_4 FlhDC + pFlhB \underset{k_{-4}}{\overset{k_4}{\rightleftharpoons}} (FlhDC)_{n_4} \bowtie pFlhB \qquad K_4^{n_4} = \frac{k_{-4}}{k_4} \tag{4}$$

$$(FlhDC)_{n_3} \bowtie pFliA \xrightarrow{b_5} FliA + (FlhDC)_{n_3} \bowtie pFliA \tag{5}$$

$$(FlhDC)_{n_4} \bowtie pFlhB \xrightarrow{b_6} FP3 + (FlhDC)_{n_4} \bowtie pFlhDC \tag{6}$$

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specific to pTet-circuit

$$(FlhDC)_{n_4} \bowtie pFlhB \xrightarrow{b_6} TetR + (FlhDC)_{n_4} \bowtie pFlhB \tag{7}$$

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specific to pFlhDC-circuit

$$(FlhDC)_{n_4} \bowtie pFlhB \xrightarrow{b_6} OmpR^* + (FlhDC)_{n_4} \bowtie pFlhB \tag{8}$$

$$n_9 FliA + pFliA \underset{k_{-9}}{\overset{k_9}{\rightleftharpoons}} (FliA)_{n_9} \bowtie pFliA \qquad K_9^{n_9} = \frac{k_{-9}}{k_9} \tag{9}$$

$$n_{10} FliA + pFlhB \underset{k_{-10}}{\overset{k_{10}}{\rightleftharpoons}} (FliA)_{n_{10}} \bowtie pFlhB \qquad K_{10}^{n_{10}} = \frac{k_{-10}}{k_{10}} \tag{10}$$

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specific to pFlhDC-circuit

$$n_{11} FliA + pFlhDC \underset{k_{-11}}{\overset{k_{11}}{\rightleftharpoons}} (FliA)_{n_{11}} \bowtie pFlhDC \qquad K_{11}^{n_{11}} = \frac{k_{-11}}{k_{11}} \tag{11}$$

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specific to pFlhDC-circuit

$$(FliA)_{n_{11}} \bowtie pFlhDC \xrightarrow{b_{12}} FlhDC + (FliA)_{n_{11}} \bowtie pFlhDC \tag{12}$$

$$(FliA)_{n_9} \bowtie pFliA \xrightarrow{b_{13}} FliA + (FliA)_{n_9} \bowtie pFliA \tag{13}$$

$$(FliA)_{n_{10}} \bowtie pFlhB \xrightarrow{b_{14}} FP3 + (FliA)_{n_{10}} \bowtie pFlhB \tag{14}$$

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specific to pTet-circuit

$$(FliA)_{n_{10}} \bowtie pFlhB \xrightarrow{b_{14}} TetR + (FliA)_{n_{10}} \bowtie pFlhB \tag{15}$$

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specific to pFlhDC-circuit

$$(FliA)_{n_{10}} \bowtie pFlhB \xrightarrow{b_{14}} OmpR^* + (FliA)_{n_{10}} \bowtie pFlhB \tag{16}$$

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specific to pTet-circuit

$$n_{17}aTc + TetR \underset{k_{-17}}{\overset{k_{17}}{\rightleftharpoons}} (aTc)_{n_{17}} \bowtie TetR \qquad K_{17} = \frac{k_{-17}}{k_{17}} \tag{17}$$

specific to pTet-circuit

$$n_{18}TetR + pTet \underset{k_{-18}}{\overset{k_{18}}{\rightleftharpoons}} (TetR)_{n_{18}} \bowtie pTet \qquad K_{18}^{n_{18}} = \frac{k_{-18}}{K_{18}^{n_{18}}} \tag{18}$$

specific to pFlhDC-circuit

$$n_{19}OmpR^{*} + pFlhDC \underset{k_{-19}}{\overset{k_{19}}{\rightleftharpoons}} (OmpR^{*})_{n_{19}} \bowtie pFlhDC \qquad K_{19}^{n_{19}} = \frac{k_{-19}}{k_{19}} \tag{19}$$

$$FlhDC \xrightarrow{\gamma_{20}} \emptyset \tag{20}$$

$$FliA \xrightarrow{\gamma_{21}} \emptyset \tag{21}$$

$$FP3 \xrightarrow{\gamma_{22}} \emptyset \tag{22}$$

specific to pTet-circuit

$$TetR \xrightarrow{\gamma_{23}} \emptyset \tag{23}$$

specific to pTet-circuit

$$(aTc)_{n_{17}} \bowtie TetR \xrightarrow{\gamma_{24}} \emptyset \qquad K_{17}^{eff} = K_{17} + \frac{\gamma_{24}}{k_{17}} \tag{24}$$

specific to pFlhDC-circuit

$$OmpR^{*} \xrightarrow{\gamma_{25}} \emptyset \tag{25}$$

$$(3) \Rightarrow \qquad [(FlhDC)_{n_3} \bowtie pFliA]_{eq} = \frac{[FlhDC]^{n_3}}{K_3^{n_3} + [FlhDC]^{n_3}} \cdot [pFliA^{total}] \tag{26}$$

$$(4) \Rightarrow \qquad [(FlhDC)_{n_4} \bowtie pFlhB]_{eq} = \frac{[FlhDC]^{n_4}}{K_4^{n_4} + [FlhDC]^{n_4}} \cdot [pFlhB^{total}] \tag{27}$$

$$(9) \Rightarrow \qquad [(FliA)_{n_9} \bowtie pFliA]_{eq} = \frac{[FliA]^{n_9}}{K_9^{n_9} + [FliA]^{n_9}} \cdot [pFliA^{total}] \tag{28}$$

$$(10) \Rightarrow \qquad [(FliA)_{n_{10}} \bowtie pFlhB]_{eq} = \frac{[FliA]^{n_{10}}}{K_{10}^{n_{10}} + [FliA]^{n_{10}}} \cdot [pFlhB^{total}] \tag{29}$$

specific to pFlhDC-circuit

$$(11) \Rightarrow \qquad [(FliA)_{n_{11}} \bowtie pFlhDC]_{eq} = \frac{[FliA]^{n_{11}}}{K_{11}^{n_{11}} + [FliA]^{n_{11}}} \cdot [pFlhDC^{total}] \tag{30}$$

specific to pTet-circuit

$$\begin{matrix}(17)\\(24)\end{matrix} \Rightarrow \begin{cases} [TetR^{free}] = [TetR^{total}] - [(aTc)_{n_{17}} \bowtie TetR] \\ \left([aTc]_i - [(aTc)_{n_{17}} \bowtie TetR]_{eq}\right)^{n_{17}} \left([TetR^{total}] - [(aTc)_{n_{17}} \bowtie TetR]_{eq}\right) - K_{17}^{eff}[(aTc)_{n_{17}} \bowtie TetR]_{eq} = 0 \\ 0 < n_{17}[(aTc)_{n_{17}} \bowtie TetR]_{eq} < [aTc]_i \qquad ; \qquad 0 < [(aTc)_{n_{17}} \bowtie TetR]_{eq} < [TetR^{total}] \end{cases} \tag{31}$$

specific to pTet-circuit

$$(18) \Rightarrow \qquad [pTet]_{eq} = \frac{K_{18}^{n_{18}}}{K_{18}^{n_{18}} + [TetR^{free}]^{n_{18}}} \cdot [pTet^{total}] \tag{32}$$

specific to pFlhDC-circuit

$$(19) \Rightarrow \qquad [pFlhDC]_{eq} = \frac{K_{19}^{n_{19}}}{K_{19}^{n_{19}} + [OmpR^*]^{n_{19}}} \cdot [pFlhDC^{total}] \tag{33}$$

specific to pTet-circuit

$$\begin{matrix}(1)\\(20)\end{matrix} \Rightarrow \frac{d[FlhDC]}{dt} = b_1[pTet]_{eq} - \gamma_{20}[FlhDC] \tag{34a}$$

$$\begin{matrix}(31)\\(32)\end{matrix} \Rightarrow \frac{d[FlhDC]}{dt} = \beta_1 \cdot \frac{K_{18}^{n_{18}}}{K_{18}^{n_{18}} + [TetR^{free}]^{n_{18}}} - \gamma_{20}[FlhDC] \tag{34}$$

specific to pFlhDC-circuit

$$\begin{matrix}(2)\\(12)\\(20)\end{matrix} \Rightarrow \frac{d[FlhDC]}{dt} = b_2[pFlhDC^{free}]_{eq} + b_{12}[(FliA)_{n_{11}} \bowtie pFlhDC]_{eq} - \gamma_{20}[FlhDC] \tag{35a}$$

$$\begin{matrix}(30)\\(33)\end{matrix} \Rightarrow \frac{d[FlhDC]}{dt} = \frac{K_{19}^{n_{19}}}{K_{19}^{n_{19}} + [OmpR^*]^{n_{19}}} \left(\beta_2 \cdot \frac{K_{11}^{n_{11}}}{K_{11}^{n_{11}} + [FliA]^{n_{11}}} + \beta_{12} \cdot \frac{[FliA]^{n_{11}}}{K_{11}^{n_{11}} + [FliA]^{n_{11}}} \right) - \gamma_{20}[FlhDC] \tag{35}$$

$$\begin{matrix}(5)\\(13)\\(21)\end{matrix} \Rightarrow \frac{d[FliA]}{dt} = b_5[(FlhDC)_{n_3} \bowtie pFliA]_{eq} + b_{13}[(FliA)_{n_9} \bowtie pFliA]_{eq} - \gamma_{21}[FliA] \tag{36a}$$

$$\begin{matrix}(26)\\(28)\end{matrix} \Rightarrow \frac{d[FliA]}{dt} = \beta_5 \cdot \frac{[FlhDC]^{n_3}}{K_3^{n_3} + [FlhDC]^{n_3}} + \beta_{13} \cdot \frac{[FliA]^{n_9}}{K_9^{n_9} + [FliA]^{n_9}} - \gamma_{21}[FliA] \tag{36}$$

$$\begin{matrix}(6)\\(14)\\(22)\end{matrix} \Rightarrow \frac{d[FP3]}{dt} = b_6[(FlhDC)_{n_4} \bowtie pFlhB]_{eq} + b_{14}[(FliA)_{n_{10}} \bowtie pFlhB]_{eq} - \gamma_{22}[FP3] \tag{37a}$$

$$\begin{matrix}(27)\\(29)\end{matrix} \Rightarrow \frac{d[FP3]}{dt} = \beta_6 \cdot \frac{[FlhDC]^{n_4}}{K_4^{n_4} + [FlhDC]^{n_4}} + \beta_{14} \cdot \frac{[FliA]^{n_{10}}}{K_{10}^{n_{10}} + [FliA]^{n_{10}}} - \gamma_{22}[FP3] \tag{37}$$

specific to pTet-circuit

$$\begin{matrix}(7)\\(15)\\(23)\end{matrix} \Rightarrow \frac{d[TetR]}{dt} = b_6[(FlhDC)_{n_4} \bowtie pFlhB]_{eq} + b_{14}[(FliA)_{n_{10}} \bowtie pFlhB]_{eq} - \gamma_{23}[TetR] \tag{38a}$$

$$\begin{matrix}(27)\\(29)\end{matrix} \Rightarrow \frac{d[TetR]}{dt} = \beta_6 \cdot \frac{[FlhDC]^{n_4}}{K_4^{n_4} + [FlhDC]^{n_4}} + \beta_{14} \cdot \frac{[FliA]^{n_{10}}}{K_{10}^{n_{10}} + [FliA]^{n_{10}}} - \gamma_{23}[TetR] \quad (38)$$

Solve then eqn.(31) *to get* $[TetR^{free}]$ *in function of* $[TetR^{total}] := [TetR]$

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specific to pFlhDC-circuit

$$\begin{matrix}(8)\\(16)\\(25)\end{matrix} \Rightarrow \frac{d[OmpR^*]}{dt} = b_6[(FlhDC)_{n_4} \bowtie pFlhB]_{eq} + b_{14}[(FliA)_{n_{10}} \bowtie pFlhB]_{eq} - \gamma_{25}[OmpR^*] \quad (39a)$$

$$\begin{matrix}(27)\\(29)\end{matrix} \Rightarrow \frac{d[OmpR^*]}{dt} = \beta_6 \cdot \frac{[FlhDC]^{n_4}}{K_4^{n_4} + [FlhDC]^{n_4}} + \beta_{14} \cdot \frac{[FliA]^{n_{10}}}{K_{10}^{n_{10}} + [FliA]^{n_{10}}} - \gamma_{25}[OmpR^*] \quad (39)$$

Added Equations for the "*FIFO module*"

$$n_3 FlhDC + pFliL \underset{k_{-40}}{\overset{k_{40}}{\rightleftharpoons}} (FlhDC)_{n_{40}} \bowtie pFliL \qquad K_{40}^{n_{40}} = \frac{k_{-40}}{k_{40}} \quad (40)$$

$$n_{41} FliA + pFliL \underset{k_{-41}}{\overset{k_{41}}{\rightleftharpoons}} (FliA)_{n_{41}} \bowtie pFliL \qquad K_{41}^{n_{41}} = \frac{k_{-41}}{k_{41}} \quad (41)$$

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specific to pFlgA-circuit

$$n_{42} FlhDC + pFlgA \underset{k_{-42}}{\overset{k_{42}}{\rightleftharpoons}} (FlhDC)_{n_{42}} \bowtie pFlgA \qquad K_{42}^{n_{42}} = \frac{k_{-42}}{k_{42}} \quad (42)$$

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specific to pFlgA-circuit

$$n_{43} FliA + pFlgA \underset{k_{-43}}{\overset{k_{43}}{\rightleftharpoons}} (FliA)_{n_{43}} \bowtie pFlgA \qquad K_{43}^{n_{43}} = \frac{k_{-43}}{k_{43}} \quad (43)$$

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specific to pFlgB-circuit

$$n_{44} FlhDC + pFlgB \underset{k_{-44}}{\overset{k_{44}}{\rightleftharpoons}} (FlhDC)_{n_{44}} \bowtie pFlgB \qquad K_{44}^{n_{44}} = \frac{k_{-44}}{k_{44}} \quad (44)$$

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specific to pFlgB-circuit

$$n_{45} FliA + pFlgB \underset{k_{-45}}{\overset{k_{45}}{\rightleftharpoons}} (FliA)_{n_{45}} \bowtie pFlgB \qquad K_{43}^{n_{43}} = \frac{k_{-45}}{k_{45}} \quad (45)$$

$$(FlhDC)_{n_{40}} \bowtie pFliL \xrightarrow{b_{46}} FP1 + (FlhDC)_{n_{40}} \bowtie pFliL \quad (46)$$

$$(FliA)_{n_{41}} \bowtie pFliL \xrightarrow{b_{47}} FP1 + (FliA)_{n_{41}} \bowtie pFliL \quad (47)$$

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specific to pFlgA-circuit

$$(FlhDC)_{n_{42}} \bowtie pFlgA \xrightarrow{b_{48}} FP2 + (FlhDC)_{n_{42}} \bowtie pFlgA \tag{48}$$

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specific to pFlgA-circuit

$$(FliA)_{n_{43}} \bowtie pFlgA \xrightarrow{b_{49}} FP2 + (FliA)_{n_{43}} \bowtie pFlgA \tag{49}$$

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specific to pFlgB-circuit

$$(FlhDC)_{n_{44}} \bowtie pFlgB \xrightarrow{b_{50}} FP2 + (FlhDC)_{n_{44}} \bowtie pFlgB \tag{50}$$

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specific to pFlgB-circuit

$$(FliA)_{n_{45}} \bowtie pFlgB \xrightarrow{b_{51}} FP2 + (FliA)_{n_{45}} \bowtie pFlgB \tag{51}$$

$$FP1 \xrightarrow{\gamma_{52}} \emptyset \tag{52}$$

$$FP2 \xrightarrow{\gamma_{53}} \emptyset \tag{53}$$

$$(40) \Rightarrow \qquad [(FlhDC)_{n_{40}} \bowtie pFliL]_{eq} = \frac{[FlhDC]^{n_{40}}}{K_{40}^{n_{40}} + [FlhDC]^{n_{40}}} \cdot [pFliL^{total}] \qquad (54)$$

$$(41) \Rightarrow \qquad [(FliA)_{n_{41}} \bowtie pFliL]_{eq} = \frac{[FliA]^{n_{41}}}{K_{41}^{n_{41}} + [FliA]^{n_{41}}} \cdot [pFliL^{total}] \qquad (55)$$

specific to pFlgA-circuit

$$(42) \Rightarrow \qquad [(FlhDC)_{n_{42}} \bowtie pFlgA]_{eq} = \frac{[FlhDC]^{n_{42}}}{K_{42}^{n_{42}} + [FlhDC]^{n_{42}}} \cdot [pFlgA^{total}] \qquad (56)$$

specific to pFlgA-circuit

$$(43) \Rightarrow \qquad [(FliA)_{n_{43}} \bowtie pFlgA]_{eq} = \frac{[FliA]^{n_{43}}}{K_{43}^{n_{43}} + [FliA]^{n_{43}}} \cdot [pFlgA^{total}] \qquad (57)$$

specific to pFlgB-circuit

$$(44) \Rightarrow \qquad [(FlhDC)_{n_{44}} \bowtie pFlgB]_{eq} = \frac{[FlhDC]^{n_{44}}}{K_{44}^{n_{44}} + [FlhDC]^{n_{44}}} \cdot [pFlgB^{total}] \qquad (58)$$

specific to pFlgB-circuit

$$(45) \Rightarrow \qquad [(FliA)_{n_{45}} \bowtie pFlgB]_{eq} = \frac{[FliA]^{n_{45}}}{K_{45}^{n_{45}} + [FliA]^{n_{45}}} \cdot [pFlgB^{total}] \qquad (59)$$

(46)
(47) $$\Rightarrow \frac{d[FP1]}{dt} = b_{46}[(FlhDC)_{n_{40}} \bowtie pFliL]_{eq} + b_{47}[(FliA)_{n_{41}} \bowtie pFliL]_{eq} - \gamma_{52}[FP1] \tag{60a}$$
(52)

(54)
(55) $$\Rightarrow \frac{d[FP1]}{dt} = \beta_{46} \cdot \frac{[FlhDC]^{n_{40}}}{K_{40}^{n_{40}} + [FlhDC]^{n_{40}}} + \beta_{47} \cdot \frac{[FliA]^{n_{41}}}{K_{41}^{n_{41}} + [FliA]^{n_{41}}} - \gamma_{52}[FP1] \tag{60}$$

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specific to pFlgA-circuit

(48)
(49) $$\Rightarrow \frac{d[FP2]}{dt} = b_{48}[(FlhDC)_{n_{42}} \bowtie pFlgA]_{eq} + b_{49}[(FliA)_{n_{43}} \bowtie pFlgA]_{eq} - \gamma_{53}[FP2] \tag{61a}$$
(53)

(56)
(59) $$\Rightarrow \frac{d[FP2]}{dt} = \beta_{48} \cdot \frac{[FlhDC]^{n_{42}}}{K_{42}^{n_{42}} + [FlhDC]^{n_{42}}} + \beta_{49} \cdot \frac{[FliA]^{n_{43}}}{K_{43}^{n_{43}} + [FliA]^{n_{43}}} - \gamma_{53}[FP2] \tag{61}$$

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(50)
(51) $$\Rightarrow \frac{d[FP2]}{dt} = b_{50}[(FlhDC)_{n_{44}} \bowtie pFlgB]_{eq} + b_{51}[(FliA)_{n_{45}} \bowtie pFlgB]_{eq} - \gamma_{53}[FP2] \tag{62a}$$
(53)

(58)
(59) $$\Rightarrow \frac{d[FP2]}{dt} = \beta_{50} \cdot \frac{[FlhDC]^{n_{44}}}{K_{44}^{n_{44}} + [FlhDC]^{n_{44}}} + \beta_{51} \cdot \frac{[FliA]^{n_{45}}}{K_{45}^{n_{45}} + [FliA]^{n_{45}}} - \gamma_{53}[FP2] \tag{62}$$